The Dragged Surge Motion of Tension-Leg Type Fish Cage System

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ABSTRACT

In this study a typical tension-leg type net cage system is presented, and a set of equations of motion for the cage system of simplified two-dimensional case are derived. Subsequently a close form solution is obtained when the wave-structure interaction between the top buoyant structure and the incident wave and the drag motion of the tether and net are both considered. The parameter of solidity as a ratio of the twine diameter and the mesh size of the cage is also taken into account in the analysis. The purpose of this study is to investigate the surge motion for the tension-leg type cage system when subjected to incident waves and flow drags on the net-cages, and furthermore, to compare the cage behavior with the case when the net-cage drag is not considered. Numerical examples are carried out and the results discussed focusing on the cage motion when subjected to net-cage drags in the waves of a range of periods. It is found that there is significant influence of net-cage drag on the surge motion of the net-cage system. Under various parameters such as the material property, twine diameter, mesh size and the draft of the collar structure, the surge motion of the net-cage system were also studied and the behavior is generally similar to the case without drag effect.

張力繫纜式箱網之拖曳徘徊運動

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摘要

在本研究中將探討一繫纜式箱網結構，於海中受到波浪作用時之拖曳徘徊運動。首先在考慮自由液面、繩網拖曳及結構與波浪互制情況下，建立一簡化之二維模式系統方程式，作為繫纜式箱網於海中受力時之分析模式，其中並考慮由箱網網片所產生之遮蔽效應。分析時，在考慮波浪與大型結構之互制作用與繩網拖曳效應時，得出箱網及其繫纜運動之解析解。其目的在瞭解繫纜式箱網於海中受風浪作用時，箱網變形及繩網之受力行為，及其與使用材料、網目、網徑大小等之間的關係。根據分析結果發現，在考慮箱網及繩網的拖曳效應下，箱網及上部浮體結構運動時之振幅將大為降低；而除了波浪的影響之外，箱網之動力行為與使用之繩絨材料、網袋深度等有密切關係，另外亦受到網纜直徑與網目比值之影響。

1. Introduction

This paper provides a study on the dragged surge motion of tension-leg type fish farming cage system in the marine environment. It is well known that due to massive over-pumping of the ground water for the aquarium along the south-west coastal area of Taiwan, the induced continuous ground settlement has become a serious problem (CSWSR Report, 1994). This uncontrollable ground settlement not only shrinks west coastal line of Taiwan but also sinks many villages during the storming season. The cost has been heightened for the aquarium operated along the coastal line and thus it gets more difficult for the aquaculture
industry. One method that can lengthen the aquarium fishery and also reduce the impact on the coastal line is the cage fish farming to move the aquarium into the sea. Cage fish farming is one of the methods that have been widely used to rear the fish in the sea by using a net-cage system.

Many fish farming cage systems were proposed (Hurtadoponce 1992, Edward et al. 1988, Yamamoto, et al. 1988). Generally, the fish farming cage is a set of netting cage system consisted of four major parts of members: namely, the buoyancy collar system, the netting cage, the mooring tether to constrain and position the cage, and the anchoring mass at the sea bed. The collar member generally combined with floaters provides the required buoyancy for the cage system. The net cage is the main structure for rearing the fish. The twine diameter, the net size, the pretension force and the overall dimension of the cage are important parameters related to the size, number and species of the fish to be reared in the net-cage. The design of the tether and the anchoring system is dependent on the requirement of aquarium, the environmental loading and the response of the cage subjected to environmental loading.

The tension-leg type cage system is the newly developed one, which was adapted from the tension-leg platform system. A typical group of tension-leg type net-cage system was shown in Fig.1 of which part (a) is the illustration of the whole net-cage system and part (b) is the side view of the net-cage. Some studies for this type of cage system have been performed on both experiments (Fu et al. 1989, Fujita et al. 1991) and theories (Lee et al. 1997, 1999) since late 80s. The results showed that the major concern on the fish-farming net-cage system is the mooring force and the dynamic motion and the deformation (deflection) for the tether and the net-cage.

In the previous studies Lee and Wang (1997) studied a round shape net cage system by means of a numerical scheme when the interactions between the wave and the top collar structure was ignored. Wang and Lee (1998) studied a similar series of net cage system in an analytical method with considering the interactions between the wave and top collar structure and also the net cages while the drag effect of the net-tether was ignored. It was found that the influence of the net cage on the flow field is not as significant as expected if the mesh size is not too small. Therefore, the drag effect is being concerned at this stage of analysis although it is difficult to solve the flow field, platform motion and the net-tether motion simultaneously.

In this study, by considering the drag effect and also the free surface in between the collar member, the equation of motion and the corresponding solution for both the net-cage and the tethers of the tension-leg fish farming cage system were derived and solved analytically. The net-cage is subjected to flow-induced drag motion and the top collar subjected to wave-induced surge motion. The material property and the mechanical behavior of the tether were both taken into accounts. Based on the previous studies, (Lee & Lee 1993, Lee et al. 1997, 1999, Wang & Lee 1998, and Lee & Wang 1999) firstly, the equations for the scattering problem and the radiation problem were established at the related boundaries with variables of velocity potential obtained from the Laplace equation. Then the forcing function related to the aforementioned velocity potentials was used in the equation of motion for the platform and the net-cages. Combined all of these equations together the unknown
coefficients and the responses of the platform were solved simultaneously in analytical method.

The purpose of this study is to investigate the surge motion for the tension-leg type cage system when subjected to incident waves and flow drags on the net-cages, and furthermore, to compare the cage behavior with the case in which the net-cage drag is not considered. Numerical examples are carried out and the results discussed focusing on the cage motion when subjected to net-cage drags in the waves of a range of periods. It was found that there is significant influence of net-cage drag on the surge motion of the net-cage system. Under various parameters such as the material property, twine diameter, mesh size and the net-cage size, the surge motion of the net-cage system was also studied and the behavior was quite different from the case without drag effect.

2. General Wave Theory

For the inviscid and incompressible fluid and irrotational flow, a single-valued velocity potential \( \phi \) can be defined as \( \vec{u} = -\nabla \phi \), where \( u \) is the velocity potential and \( \nabla \) is the gradient operator. The velocity potential satisfies the Laplace equation: \( \nabla^2 \phi = 0 \) and the Bernoulli equation in the flow field as

\[
-\frac{\partial \phi}{\partial t} + \frac{1}{2} \nabla \phi \cdot \nabla \phi + \frac{P}{\rho} + \phi g z = 0
\]

where \( P \) is the pressure, \( \rho \) the water density, \( g \) the gravitational constant, \( z \) the variable of depth. The nonlinear term in equation (1) was ignored when the linear small amplitude wave was assumed.

A two-dimensional tension leg net-cage system interacting with a monochromatic small amplitude wave propagating in the \(+x\)-direction was considered here as was shown in Fig.2. The waveform and the associated velocity potential are given accordingly as,

\[
\eta_i = -iA_i e^{-(K_i x+i\alpha t)}
\]

(2)

and

\[
\phi_i = \frac{A_i g \cos(K_i (z+h))}{\sigma \cos(K_i h)} e^{-(K_i x+i\alpha t)}
\]

(3)

where \( A_i \) is the wave amplitude, \( g \) is the gravitational constant and \( h \) is the water depth. \( \alpha = \frac{2\pi}{T} \) is the angular frequency with period \( T \). \( K_i = -ik \) and \( k = \frac{2\pi}{L} \) is the wave number with wavelength \( L \). \( K_i \) satisfies the dispersion relation given by dispersion relation: \( \sigma^2 = gK_1 \tan(K_1 h) \)

3. Tension-Leg Net-Cage System

In the tension-leg net-cage system, the motion of the structure is assumed to be small. The wave induced structural motion can be solved from the imposed boundary problem. Due to the linearity of the problem, the problem can be incorporated into a scattering and a radiation problem. The wave force calculated from the scattering problem provides the force function in the radiation problem, and the forced oscillation then generates outgoing waves.

Fig.2 Definition sketch of the problem

As was shown in Fig.2, the tension-leg net-cage system was divided into five regions. In Region I: the total velocity potential \( \phi = \phi_i + \phi_h + \phi_w \). In the other regions the total velocity potential consists of velocity potentials of scattered waves and radiated waves such as in Region II: \( \phi = \phi_{ii} + \phi_{iw} \); in Region III: \( \phi = \phi_{ii} + \phi_{iw} \); in Region IV: \( \phi = \phi_{ii} + \phi_{iw} \); and in Region V: \( \phi = \phi_{ii} + \phi_{iw} \). The subscript \( s \) denotes the scattering problem and \( w \) denotes the radiation (wave making) problem. All of the velocity potentials satisfy the Laplace equation. Furthermore, on the infinite boundary \(-\infty \) in Region I and \(+\infty \) in Region V, the Sommerfeld’s radiation condition is satisfied for unique solutions.

3.1 The Scattering and Radiation Problem

In the scattering problem, the incident wave is considered to be diffracted by a fixed structure, while in the radiation problem the net-cage is being forced into motion by the wave force induced by incident waves and scattered waves. The corresponding boundary conditions for both the scattering and the radiation problem were shown in Fig.3.
Fig. 3 Boundary conditions (s: scattering problem, w: radiation problem)

Applying the method of the separation of variables, matching the horizontal boundary conditions in each region and applying the Sommerfeld's condition to regions I and V, the velocity potential in each region can be found as:

**Region I and Region V:**
\[ \Phi_{s1/w} = \sum_{n=1}^{\infty} \frac{A_{s1/g}}{\sigma} \frac{\cos[K_n(z+h)]}{\cos(K_nh)} e^{i(K_n(x+b)+\sigma t)} \]  
\[ \Phi_{s1/w} = \sum_{n=1}^{\infty} \frac{A_{s1/g}}{\sigma} \frac{\cos[K_n(z+h)]}{\cos(K_nh)} e^{-iK_n(x+b)-\sigma t} \]

where the eigenvalues $K_n$ can be solved by the dispersion equation: $\sigma^2 = -gK_n \tan(K_nh)$.

**Region II:**
\[ \Phi_{s2/w} = \frac{i\sigma}{\sigma} [A_{s2/g} \frac{x}{l+b} + \frac{A_{s2/w} \cos(K_n(z+h)) + \sum_{n=2}^{\infty}(1-l)^{n-1}}{m=2} e^{iK_n(x+b)+\sigma t} \]
\[ \Phi_{s2/w} = \frac{i\sigma}{\sigma} [A_{s2/g} \frac{x}{l+b} + \frac{A_{s2/w} \cos(K_n(z+h)) + \sum_{n=2}^{\infty}(1-l)^{n-1}}{m=2} e^{-iK_n(x+b)-\sigma t} \]

**Region III:**
\[ \Phi_{s3/w} = \frac{i\sigma}{\sigma} [A_{s3/g} \frac{x}{l+b} + \frac{A_{s3/w} \cos(K_n(z+h)) + \sum_{n=2}^{\infty}(1-l)^{n-1}}{m=2} e^{iK_n(x+b)+\sigma t} \]
\[ \Phi_{s3/w} = \frac{i\sigma}{\sigma} [A_{s3/g} \frac{x}{l+b} + \frac{A_{s3/w} \cos(K_n(z+h)) + \sum_{n=2}^{\infty}(1-l)^{n-1}}{m=2} e^{-iK_n(x+b)-\sigma t} \]

**Region IV:**
\[ \Phi_{s4/w} = \frac{i\sigma}{\sigma} [A_{s4/g} \frac{x}{l+b} + \frac{A_{s4/w} \cos(K_n(z+h)) + \sum_{n=2}^{\infty}(1-l)^{n-1}}{m=2} e^{iK_n(x+b)+\sigma t} \]
\[ \Phi_{s4/w} = \frac{i\sigma}{\sigma} [A_{s4/g} \frac{x}{l+b} + \frac{A_{s4/w} \cos(K_n(z+h)) + \sum_{n=2}^{\infty}(1-l)^{n-1}}{m=2} e^{-iK_n(x+b)-\sigma t} \]

where similarly $K_{th}$ may be obtained from the dispersion equation while the undetermined series of coefficients must be obtained from the kinetic and dynamic boundary conditions as was shown in Fig. 3.

### 3.2 Equation of Motion of the Top Structure

The equation of motion for the top collar structure is presented as
\[ M \frac{d^2 \xi}{dt^2} + K^* \xi = F_wx + F_{Dx1} + F_{Dx2} + F_{Nz1} + F_{Nz2} \]

where $K^*$ is the equivalent stiffness of the system related to the pretension force of the tether and the structure displacement is assumed to be in accordance with the wave induced surge motion as $\xi = \xi(t)$.

The wave force is given by
\[ F_w = \rho \frac{\partial}{\partial t} \left( \frac{\partial \Phi}{\partial t} \right) + \frac{\partial \Phi}{\partial t} \]
The drag force on the net-tether is obtained through the Morison's equation as

\[ F_{\text{D,net}} = \int_0^L \left[ 0.5 \rho_s D_C |u - x'| (u - x') + 0.25 \rho_s D_i C_m (u - x') \right] dz \]

and the drag force on the net cage is presented as

\[ \mathcal{F}_{\text{net}} = \int_0^L \left[ 0.5 \rho_s D_C |u - x'| (u - x') + \rho_s C_m V (u - x') \right] dz \]

where \( d_i \) is the draft of the collar structure, \( d_b \) is the depth of the bottom of cage, \( C_m \) is the coefficient of the added mass, \( V \) is the unit volume of the net and \( C_m \) is the drag coefficient of the net. The drag coefficient of the net is obtained from an empirical value related to the twine diameter, net size and directional angle of the flow.

3.3 Equation of Motion of Net-cages

The equation of motion for the tether and net-cage is given by

\[ \rho_* \frac{\partial^2 x}{\partial t^2} + C_s \frac{\partial x}{\partial t} - T^* \frac{\partial^2 x}{\partial z^2} = f_{D,\text{in}} (z, t) + f_{\text{net}} (z, t) \]

where \( \rho_* \) is the mass of unit length of tether-net, \( T^* \) is the pretension force of the tether. The forces in the right hand side including the drags on nets and the tethers are both from the modified Morison's equation with a similar form as equation (11) and (12) except for the integration. They were presented as follows:

\[ f_{D,\text{in}} = 0.5 \rho_s D_C |u - x'| (u - x') + 0.25 \rho_s D_i C_m (w' - x') \]

and

\[ f_{\text{net}} = 0.5 \rho_s D_C |u - x'| (u - x') + \rho_s C_m V (u - x') \]

Assuming \( x(z, t) = Z(z)e^{-igt} \) and setting up boundary conditions and continuity condition for the displacement and the pretension force, the equation can be solved. By combining the solutions of \( x \) and velocity potential and substituting back into the equation of motion for the collar structure, equation (9) can be solved for the motion of the structural motion and then the motion of the tether-net and the velocity potential simultaneously.

4. Response of the Net-Cage Subjected to Waves

Numerical examples for the tension-leg fish-farming net-cage system were carried out in this section. The investigation was focused on the deformation of the net-cage, the displacement of the tether and the response of top structure when subjected to waves with various periods. The influence of the twine diameter, mesh size and the dimension of the net-cage on both the platform (collar member) motion and the mooring tether was also examined. Before numerical analysis the convergence of the analytical solution was examined and verified first.

4.1 The convergence of the analytical solution

Since the solution of the response in this study is in analytical form, the convergence test was performed for the reflection coefficient in terms of the mode number needed for a stable solution of series form. Presented in Fig.4 is the reflection coefficient corresponding to the mode number when the wave period is ranged from 2 second to 10 second. It is observed that when the mode number is more 10 the reflection coefficient will converge to a constant value.

4.2 The effect of twine diameter

Fig.5 showed the dimensionless amplitude of the collar member corresponding to the wave frequency while the twine diameter is varied. Corresponding to the increase of the twine diameter the response was reduced in the same wave conditions. This result seems to be in accordance to the tether-net responses.

4.3 The effect of mesh size

Besides the difference in the twine diameter of the net-cage system, the mesh size is also an important factor to the cage response. Thus the mesh size of the net-cage was examined here. Fig.6 showed the response of a net-cage system with varied mesh size of the net-cage, where it showed that compared to the case without net-cage the cage system with smaller mesh size has smaller response on the top collar member. This is probably due to the increase of the drag effect of the net-cage system, thus reducing the response of the collar member in general.

4.4 The effect of the dimension of the net-cage

It is also concerned that if the increase of the dimension of the net-cage will influence the motion of the cage system. It is clear that the increase in the width will reduce the response. Therefore, the dimension of the cage depth is the interest of study. Fig.7 presented the response of the top collar of net-
Fig. 4 Convergence test on the reflection factor.

Fig. 5 Dimensionless amplitude of the collar member corresponding to twine diameter variation.

Fig. 6 Dimensionless amplitude of the collar member corresponding to mesh size variation.

Fig. 7 Dimensionless amplitude of the collar member corresponding to cage depth variation.

Fig. 8(a) Deformation profile of tether-net when net-bag is 10 m.

Fig. 8(b) Deformation profile of tether-net when net-bag is 10 m.
cage when the cage depth was varied. When the drag effect of the net-cage is considered corresponding to the increase of the cage depth the response amplitude of the collar member seems to be reduced.

4.5 The deformation profile of the net-cage

Fig. 8 (a) and (b) showed the tether-net deformation profile with the same net-bag depth of 10 m while the wave period is from 3 second to 17 second. Basically there is not much difference in the response once the wave period is over 11 second because the net-tether has been highly strained into a curve shape by the wave force nearing the resonant period. However in the low range of wave period the deformation profile for the tether-net mainly strained by the pretension force is almost linear.

5. Conclusions

In this study a set of equations for the fish farming net-cage system and the mooring tethers based on the previous studies were derived and solved in close form solution where the interactions among the wave, collar structure and tether-net and the drag effect due to the tether-net were taken into accounts. According to the results of numerical examples for a group of simulated net-cage system subjected to waves, it was found that besides the wave properties, the cage motion was related to the twine diameter and mesh size of the net-cage, material properties of the tether, and the dimension of cage.

Compared to the structure without net-cage, the amplitude of the response for cage system appeared to be reduced as soon as the net-cage was applied. This is not the case without taking account of the tether-net drag effect (Wang & Lee 1998) because the drag effect significantly reduces the response. In the consideration of material effect the increase of the material elastic modulus will reduce the response amplitude and the resonant period as well.

Corresponding to the increase of the twine diameter the response of both the collar structure and the net-tether are reduced due to the drag effect. Due to the same drag effect the decrease of the mesh size also reduces the cage responses. The variation of the cage depth appears to have little influence on the cage responses itself but the response amplitude of the collar structure reduces corresponding to the increase of cage-bag depth.

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Reference