Modeling Velocity and Vortex Induced by an Internal Solitary Wave on a Flat Bottom

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ABSTRACT

The spatial variation in the velocity field induced by an internal solitary wave (ISW) is one of the important research themes in marine science. Together with the vortex within the system, they can affect marine operations as an ISW propagating in the ocean. To model the velocity and vortex, the governing equations presently available are modified and used for the numerical experiments. This paper presents the results of laboratory experiments and numerical modeling of the velocity field induced by an ISW of depression-type propagating on a flat bottom. Laboratory observations reveal that the water particle orbits induced by a depression ISW are opposite in direction for the upper and the lower layer in the water column. Numerical results show that the vortex is clockwise in a depression ISW and counterclockwise for an elevation ISW. The maximum velocity occurs at the extrema of an ISW waveform. Based on the laboratory results, numerical model may be improved in order to enhance the accuracy of the simulation.

Keywords: Internal Solitary Wave; Orbital motion; Numerical modeling; Laboratory experiment.

孤立內波在水平底床傳遞引發渦流場之模擬

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摘要

在浩瀚海洋內部，因水體密度層化而引起的內波，是在許多大洋中普遍存在之自然現象，其中的孤立內波振幅在南中國海可達170公尺，超過海洋表面波風浪10倍。在深海的孤立內波振幅及所引發渦動大到有足夠的能量可觸及海底障礙物，造成海底泥沙物的再懸浮及輸運，影響海域工程、海軍雲台作及青礁軍事等活動；它甚至可能影響海洋生命系統及全球環境溫度的變化。本文透過理論計算孤立內波在水平底床傳遞時之渦度與流速場，並與實驗室實驗數據相比較，再修正原參數或迴歸適當公式，以提高數值模擬之精確度。本研究結果有助於未來計算孤立內波通過動力底床之進階研究。

關鍵詞：孤立內波、粒子軌跡、數值模擬、實驗室實驗

1. Introduction

Like the ISW activities in other regions of the world, it is believed that an ISW in the South China Sea (SCS) may affect offshore oil drilling operations,
nutrient pumping, and resuspension of pollutants from seabed in relatively deep water (Osborne et al., 1978). Pilot whales are also found to follow internal solitary waves in the SCS (Moore and Lien, 2007). Presently, papers describing the physical processes of vortices induced by an ISW on a variable topography are rather limited, despite many other efforts (i.e. ASIAEX) relevant to the ISW in the Luzon Strait, between Taiwan and the Philippines (Sveen et al., 2002; Guo et al., 2004). Therefore, studies on ISW propagation through the SCS, its generation mechanism, wave evolution and vortex formation will bear important implications.

Up-to-date, many oceanographers from the USA and Taiwan have conducted extensive field experiments on internal waves in the SCS using SAR imagery, ADCP and CTD. The results arising from these field studies are mostly in terms of wave amplitude, temperature variations and flow velocities. Despite some schematic diagrams depicting the orbits of water particle motion have been proposed and accepted for more than a decade, evidence had not been available from field observations nor laboratory experiments. This paper demonstrates the results of laboratory experiments for the particle motion and numerical calculations for the vortices and velocity field induced by an ISW of depression on a flat bottom. Based these results, the velocity field within an ISW propagating on a flat bottom can be clearly understood.

2. Laboratory Experiments
2.1 Experimental Apparatus

Laboratory experiments on ISW propagation were conducted using a stratified three layered fresh-brine fluid system in a steel framed wave flume of 12 m long with a cross-section of 0.7m high by 0.5m wide. Within the flume, fluid density of the upper and lower layer was 994 and 1030 kg/m³, respectively. Beyond that, another layer indicated the pycnocline where the density varied with depth from 994 to 1030 kg/m³ within this thin layer. The experimental apparatus, included two ultrasonic gages and several recording devices, are shown in Figure 1. A removable sluice gate was mounted on the right hand side (RHS) inside the flume to generate ISWs by a “collapse” mechanism (Kao et al., 1985), resulting from the overturning of the water column produced by the difference in the interface level on either side of the vertical sluice gate. In order to study the particle motion within an ISW, small plastic particles in different dimensions and densities were used. The images taken by two digital video cameras were used to calculate the orbits and water velocity of particles. Finally, the data collected from laboratory experiments were analyzed and compared with the results obtained from numerical modeling.

![Figure 1 A schematic diagram showing experimental set-up for the present study.](image)

2.2 Experimental Results

Upon processing the data recorded, the particle orbits and water velocity can be summarized in Figures 2 and 3. In these pictures, different particle diameters (e.g., red for 4mm in diameter, green for 3mm and blue for 5mm) are represented by different colors. The particle orbit in the upper layer was in rectilinear motion in the same direction of the ISW propagation; those on the interface followed the motion of the depression ISW waveform, while those in the lower layer were in opposite direction to the ISW propagation. As seen in Fig. 3, the particles above and between the interface moved in opposite directions. Overall, for an ISW propagated on a flat bottom, the variation in particle velocity recorded by
these three different particles in the upper and lower layer was not significant. The maximum velocity in the upper layer was about 11 cm/s (blue), 10 cm/s (red), and 13 cm/s (green), respectively, in the experiments. In general, laboratory results indicated the magnitude of the horizontal velocity was greater than that of the vertical velocity as a depression ISW propagating on a flat seabed.

3. Numerical Modeling

3.1 Governing Equations

By neglecting the effect of viscosity, the equations governing the fluid motion in a three-layer fluid system (i.e., the upper, pycnocline and lower layer) are summarized in terms of conservation in density and vortex (Vlasenko, 1994; Vlasenko and Hutter, 2002). With the physical parameters for the numerical modeling sketched in Figure 4, and together with the appropriate boundary conditions, the modified governing equations are:

\[ \frac{\partial \psi}{\partial t} + \frac{\partial \psi \psi_z}{\partial x} = g \frac{\partial \rho}{\partial \rho_1} \]  
\[ \frac{\partial \rho}{\partial t} + \frac{\partial \psi \rho_z}{\partial x} = 0 \]  
\[ \psi(x,z) = a v_n \text{sech}^2\left[\left(x-v_n t \right)/\lambda\right] W_n(z) \]  
\[ \psi = 0; \ \omega = 0; \ \rho_z = 0 \] at \( z = 0 \)  
\[ \psi = 0; \ \omega = 0; \ \rho_z = 0 \] at \( z = -H \)  
\[ \psi = 0; \ \omega = 0; \ \rho_z = 0 \] at \( x = \pm L \)  

where \( \psi \) is a stream function, \( \omega \) the vortex potential, \( \rho \) a density function, \( g \) acceleration due to gravity, \( H \) the total water depth, and \( L \) a reference of the horizontal axis. \( W_n \) is a characteristic function of the Sturm-Liouville equation, \( a \) is the amplitude of an ISW, \( \lambda \) the characteristic wavelength. \( v_n \) is the group speed, which can be expressed as:

\[ v_n = \frac{N_{max} H}{\sqrt{\beta_0}} \left( 1 - \frac{a \gamma}{3 \delta \beta_0} \right) \]

where \( N_{max} \) is the maximum buoyancy frequency of the laboratory experiment; \( H \) is the total depth; \( \beta_0, \gamma, \) and \( \delta \) are the relative parameters followed the water depth.
pycnocline was 0.14m thick in an experimental run in which the depth in the upper and lower layer was 0.1 and 0.4 m, respectively. In the numerical modeling, the input of vertical density distribution was the same as the physical condition of a laboratory experiment. From the same conditions of a corresponding laboratory experiment, the velocity and vortex field calculated for a depression ISW on a flat bottom are then calculated and their results displayed in Figure 6. The blue arrowheads represent the resultant fluid velocity based on the vertical and horizontal velocity components calculated. The contour plot shows the strength of the vortex which is densely distributed around the interface.

The contours of vertical and horizontal velocity given in Figure 7 indicate the maximum horizontal velocity occurs at the trough of a depression ISW, while the largest vertical velocity, either upwards or downwards, is also at the trough. However, the maximum magnitudes of the vertical components are about an order smaller than their horizontal counterparts. Similarly, Figure 8 also shows the vertical variations in the horizontal and vertical velocity as a function of the depth at the trough of an ISW of depression. Based on these results, it is obvious that the horizontal velocity in the surface layer is much larger than that in other two layers (interface and lower layer). Because the flow direction of the horizontal velocity in the upper and lower layer are opposite to each other, the elevation where the horizontal velocity is zero may exist in the proximity of the interface of an ISW.

The density variations can be calculated from Eq. (2) and the iso-density contours in a depression ISW propagating on a flat bottom is displayed in Figure 9. Upon observing the fluctuations of the different density lines plotted in this figure, the maximum range of water depth for the density variation is about 4cm in total for an ISW of depression with amplitude of 4cm. In practical sense, the vertical variations in density with time can be recorded by a thermister chain.
4. Comparisons of Results

Using the modified numerical model, its results can now be compared with that measured from laboratory experiments. As seen in Figure 10, the numerical results and laboratory data are in good agreement for the vertical distribution of horizontal and vertical velocities at the trough of an ISW of depression. Alternatively, this comparison can be drawn in a different way, as in Figure 11, in which the maximum horizontal velocity from the numerical model ($U_{\text{max}}$) are also in good agreement with that of the laboratory data ($U_{L\text{max}}$), because both almost fall onto a straight line of equal value. In a similar manner, but with some scattering, the results for the maximum vertical velocity from the numerical model $W_{\text{Nmax}}$ and laboratory results $W_{L\text{max}}$ can be found in Figure 12. Despite the slight discrepancy of the maximum vertical velocity shown in Figure 16, these comparisons between the numerical model and laboratory experiments may be used with sufficient confidence to study the velocity field in an ISW propagating on a flat bottom, because the magnitude of the vertical velocity is in more than an order less than its horizontal counterpart.
5. Conclusions

Based on the results of present study for a depression ISW propagating in a stratified fluid system on a flat bottom, the fluid motion and velocity may be divided into three groups, depending on their vertical position. Above the interface, the particle orbit is in a rectilinear motion in the same direction as the ISW propagation. Below the interface, the orbit is curvilinear and moves in opposite direction to the wave propagation. On the interface, the orbit is also curvilinear moving with the waveform of a depression ISW.

The numerical results calculated by a modified model have shown in good agreement with that of the laboratory experiments obtained by the present authors at the National Sun Yat-sen University, Kaohsiung. A vortex field can be produced above the interface of a depression ISW as it propagates in a stratified fluid system with a flat bottom. Interestingly, the interface itself may limit the mass transfer in the vertical direction between the upper and lower layer in a stratified fluid system. Along the waveform of an ISW of depression, the maximum horizontal velocity occurs in the upper layer above the trough of an ISW where the vertical velocity is zero.

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