Deflection of Aquatic Plant by Water Waves

Chi-Min Liu 1  Ray-Yeng Yang 2  Hwung-Hweng Hwung 3

1 General Education Center, Chienkuo Technology University
2,3 Department of Hydraulics and Ocean Engineering, National Cheng Kung University

ABSTRACT

A theoretical examination for the deflection of aquatic plants by water waves is presented. An integration method developed for analyzing the deflection of cantilever beams is now applied to calculate the deformation of aquatic plants. For uniform currents, it is easy to do the calculation since the direction of the fluid loading is perpendicular to the original position of plants. For water wave cases, an extra dependence, however, appears because the wave loading has two spatial dependences. This extra spatial dependence results in difficulties of the derivation of the exact representation of the relation between the bending moment and the external loading. Therefore the integration method is modified to calculate the theoretical as well as numerical analyses. Results for different characteristics of plants and waves presented in this paper are a preliminary step for more complex flow conditions.

Keywords: Deflection; Aquatic plant; Water waves; Integration method

1.Introduction

Recently, aquatic plants attract a great deal of attention from coastal engineers due to their multi-roles in ecosystem and coastal protection. Coastal plants have many significant functions including fishery, biodiversity and coastal protection. They also have a wide variety of specific properties. For ocean engineers, the most important information may be the physical property of plants. These properties include diameter, total length, module of elasticity, and moment of inertia of plants which play an important role in calculating the plant deflection due to external forces. To simulate the deflection of plants, the most common method is to apply the relation (Timoshenko and Gere, 1961)

\[
\frac{d^2x}{dy^2} = \frac{M}{EI},
\]

where \( E \) is the module of elasticity, \( I \) the moment of inertia and \( M \) the bending moment. Equation (1)
is the well-known relation between the bending moment and the deformation of a cantilever beam. It can be also applied to the case considered herein. It is noted that the moment $M$ is a function of one or two spatial parameters for the uniform flow and the water wave cases, respectively.

A great number of theories for large deflections of cantilever beams have been developed. Bisshopp and Drucker (1945) presented an elliptic integral method which is still popular at present. Numerical integration approaches with iterative shooting techniques were derived by Lee (2001) and Magnusson et al. (2001). The finite difference and finite element methods were also studied by Golley (1997) and Kooi and Kuipers (1984), respectively. However, Dado and Al-sadder (2005) pointed out that aforementioned methods exist some drawbacks including the applicability, complexity and stability.

To this end, they presented another approach using a polynomial function. Later, A new integral approach was studied by Chen (2010) which provides a more efficient method to calculate the large deflection of cantilever beams. Chen et al. (2010) also applied the method to study uniform flow cases.

In this paper, Chen’s mathematical method (2010) is modified to analyze the water wave problem. The organization of the paper is as follows. The modified version for water-wave cases is derived in Section 2. Some numerical results are shown in Section 3 and a preliminary conclusion is made in Section 4.

2. Deflection of Aquatic Plants for Water-wave Cases

A mathematical method for solving the deflection of aquatic plants affected by water waves is derived herein. The flow field considered is depicted in Fig.1. A sine wave of frequency $\sigma$, wave number $k$ and amplitude $a$ propagates along the $x$ coordinate. A single plant of diameter $d$ and length $l_i$ is submerged in water of depth $h$. Horizontal and vertical velocities of water particles are

$$u = a\sigma \frac{\cosh ky}{\sinh kh} \cos(kx - \sigma t),$$

$$v = a\sigma \frac{\sinh ky}{\sinh kh} \sin(kx - \sigma t).$$

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It is note that all effects of plant on the flow field are neglected. Now the drag and friction forces on the plant are represented as

$$dF_x = \frac{1}{2} \rho \pi d C_j \left( u \cos \theta - v \sin \theta \right)^2 \, ds,$$

$$dF_y = \frac{1}{2} \rho \pi d C_f \left( u \sin \theta + v \cos \theta \right)^2 \, ds,$$

where $C_j$ and $C_f$ are the drag and friction coefficients, $\rho$ the fluid density, $s$ the arc length of the plant and $\theta$ the angle between the friction force and the $y$ direction. Hence, the bending moment on the plant is

$$M(x,y) = \int_{\gamma} dF_x \cdot y \cos \theta + \int_{\gamma} dF_y \cdot x \sin \theta - \int_{x} dF_x \cdot x \cos \theta + \int_{y} dF_y \cdot y \sin \theta.$$

Note that only one spatial dependence exists in Eq.(6) because the configuration of plant gives the relation $x = f(y)$. Now we apply the integration method (Chen 2010) to calculate the deflection of plant. Firstly the slope of plant is defined

$$z = \frac{dx}{dy},$$

and is then substituted to Eq.(1), the result is
where \( E \) is the elastic module and \( I \) the moment of inertia. By integrating Eq.(8), a shape function \( G \) is defined

\[
G(x,y) = \frac{z}{\sqrt{1 + z^2}} = \int_0^y \frac{M(x,y)}{E I} dy.
\]

which is equivalent to \( \sin \theta \).

Since the fluid-structure interaction is a coupled system, a standard set of procedures for numerically simulating the plant motion is required. For a fixed time \( t \), the first step is to “guess” the initial shape of plant. It may be assumed

\[
x = m y^n,
\]

where \( m \) and \( n \) are arbitrarily given. Second, using the initial guess of plant shape one can calculate the bending moment by Eq.(6). Third, the calculated bending moment is then substituted into Eq.(9) to determine the new shape of plant. Now one can repeat the second and third steps to find the final deflection of plant. During the calculation, the length of plant

\[
s(l) = \int \left[1 + \left( \frac{dx}{dy} \right)^2 \right]^{\frac{1}{2}} dy,
\]

must be checked to make sure \( s(l) = l_o \).

3. Results

Based on the method introduced in previous section, some numerical results are presented in this section. The cases considered neglect the effects of friction (\( C_f = 0 \)). Before the simulation begins, dimensionless variables and parameters are defined as

\[
\begin{align*}
Y &= \frac{Y}{h}, & X &= \frac{X}{h}, & T &= \pi \sigma, & \Pi &= \frac{a}{\alpha} \\
U &= \frac{u}{u_0}, & V &= \frac{v}{u_0}, & D &= \frac{d}{h}, & L_o &= \frac{l_o}{h},
\end{align*}
\]

where \( u_o = a \sigma \coth \mu h \). Moreover, another two parameters are also defined as

\[
\mu = \frac{kh}{h},
\]

\[
\alpha = \frac{E l}{\frac{1}{2} \rho C_f u_o^3 h^3},
\]

in which \( \mu \) represents the dispersive effect and \( \alpha \) the physical properties of plant. For the sake of simplification, the friction force is neglected and the dimensionless bending moment is expressed as

\[
M^* = \int Y (U \cos \theta - V \sin \theta) dY,
\]

\[
+ \int X (U \cos \theta - V \sin \theta) dX,
\]

and the shape function becomes

\[
G^* = \int_0^1 \frac{M^*}{\alpha} dY.
\]

Consider the case \( T = 0 \), \( \Pi = 0.1 \), \( L_o = 0.9 \), and \( D = 0.01 \), the deflection of plant is depicted as

where dash and solid lines are for cases of \( \mu = 1 \) and \( \mu = 0.1 \), respectively. It is clear that a more flexible of plant (smaller \( E \) and \( \alpha \) ) results in a larger

Fig.2 The relation between \( \delta \) and \( \theta \)

Fig.3 Deflection of cases of various \( \alpha \)
deflection. Moreover, the deflection for longer wave (smaller $\mu$) is larger than that of shorter waves, or plants in the shallower water are more “flexible” than those in deeper water. Note that the horizontal and vertical scale are different in Fig.3 which may result in a wrong understanding of the total length of plant. Cases for a fixed $\alpha$ are calculated as

![Fig.4 Deflection for cases of various $\mu$](image)

Results are similar to those shown in Fig.3 indicating the smaller $\mu$ yields a larger deflection.

Finally, the calculating time by computer is another issue in this study. It unusually depends on the value of $\alpha$. The smaller $\alpha$ (more flexible plant) results a longer calculating time. Physical parameters for Russia olive (Chen 2011) are of $d=1.35$ mm, $l_0=9$ cm and $E=5.2\times10^8$ Pa. For regular waves in a 3-m water, the value of $\alpha$ is $O(10^{-1})$ which requires a much longer computer time than cases examined herein. This weakness must be overcome in the future.

4. Conclusions

In this paper, a single aquatic plant encountering regular waves is examined. The method used is extended from Chen’ work (2010) for cases in uniform flow. For water waves, two spatial variables make the calculation more difficult and complex. Cases without friction effects are examined. It is found that plants in shallower water (longer-wave conditions) have a larger deflection than those in deeper water (shorter-wave conditions). Finally, cases with a smaller elastic modulus need a longer calculating time which must be overcome in the future. Studies on nonlinear waves are also expected.

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References